ISSN 2413-6212 (Online), ISSN 2227-1252 (Print)
Мости та тунелі: теорія, дослідження, практика, 2025, № 27

МОСТИ ТА ТУНЕЛІ: ТЕОРІЯ, ДОСЛІДЖЕННЯ, ПРАКТИКА

УДК [624.21:625.1]:519.6

O. L. TIUTKIN[1], O. I. DUBINCHYK[2*], QUAN ZHANG[3], KAIQIAN DU[4]

[1]Department «Transport Infrastructure», Ukrainian State University of Science and Technologies,
Lazariana St., 2, Dnipro, Ukraine, 49010, tel. +38 (066) 290 45 18,
e-mail o.l.tiutkin@ust.edu.ua, ORCID 0000-0003-4921-4758
[2*]Department «Transport Infrastructure», Ukrainian State University of Science and Technologies,
Lazariana St., 2, Dnipro, Ukraine, 49010, tel. +38 (095) 582 14 88,
e-mail o.i.dubinchyk@ust.edu.ua, ORCID 0000-0003-4059-2357
[3]Architectural Engineering Institute, Zhejiang Guangsha Vocational and Technical University of Construction,
Dongyang, Zhejiang, People's Republic of China, 322100, tel. +86-0579-86668872,
email 55846673@qq.com, ORCID 0009-0009-4691-9913
[4]Architectural Engineering Institute, Zhejiang Guangsha Vocational and Technical University of Construction,
Dongyang, Zhejiang, People's Republic of China, 322100, tel. +86-0579-86668872,
email dukaiqian1@163.com, ORCID 0009-0007-4977-5842

NUMERICAL AND MODAL ANALYSIS OF THE RAILWAY BRIDGE PIER

Purpose. The purpose of the scientific research is to conduct numerical and modal analyses of the railway bridge pier with a variation of load combinations and to determine the strength parameters and dynamic characteristics (shape and frequency of natural vibrations). **Methodology.** Pile foundations are the most economical and effective for bridge construction. In this case, the pile foundation of the overpass pier is constructed on bored piles. To reproduce the features of the pier's pile foundation, a spatial FE-model was used, which most accurately reflects its interaction with the surrounding massif. **Findings**. An analysis of the results of the stress-strain state of the overpass pier foundation was performed, taking into account the train load, which made it possible to obtain a conclusion about the high bearing capacity of all parts of the "pile – pile cap" system for all considered types of combinations with a safety margin of 8 and 7 times, respectively, which indicates normal operation in the future provided an unchanged state of engineering and geological conditions and loads. The connection point of the piles and the pile cap, made of concrete class C25/30, fully withstands all types of loads presented in the combinations. A modal analysis of the foundations of the overpass pier models was performed and the frequencies and shapes of the foundation were obtained, which are equal to 4.2 Hz (main tone) … 17.4 Hz, and from comparing these frequencies with the liquefaction frequencies of wet sands (30 … 50 Hz) it is clear that the support's own vibrations cannot cause vibration liquefaction. **Originality**. The originality of the study lies in obtaining the values of the stresses of the overpass pier in the static formulation, as well as the dynamic characteristics (shape and frequency of natural vibrations) in the dynamic formulation. **Practical value.** The practical value lies in the substantiated assessment of the values of the pier's vibration frequencies, which is based on the results of modal analysis and indicates the absence of sand liquefaction.

Keywords: railway bridge; pier; numerical analysis; modal analysis; modes and frequencies of natural vibrations

Introduction

The gradual transition of railways to high-speed and higher-speed traffic increases the relevance of the stress-strain state (SSS) studies of bridge (overpass) elements under train loads. At the same time, the range of studies should be somewhat increased in the case of analysis of stresses and displacements of the foundations of bridge crossings piers, since the dynamic impact of a train has specific characteristics in terms of the propagation of vibration waves within the embankment and their transition into the pier.

When analyzing the operation of bridge support structures and their foundations, it should be taken into account that they interact with the soil base and it is the features of this interaction that dictate the formation. It should be taken into account that in the general case, SSS is formed during dynamic impact, which in a simplified form can be modeled as a set of static states. However, complete information about the pile foundations of bridge supports can be obtained only when solving a comprehensive problem formulation – addressing both

ISSN 2413-6212 (Online), ISSN 2227-1252 (Print)
Мости та тунелі: теорія, дослідження, практика, 2025, № 27

static (Dubinchyk, Bannikov, Kildieiev, & Kharchenko, 2020) and dynamic problems.

The solution of the static problem allows for the assessment of the correctness of the adopted engineering solution, i.e. to analyze the stresses and displacements of the designed pier foundation under the action of its own weight and train load (Lu, Xie, & Shao, 2012; Загоруйко, & Лозовий, 2012; Дубінчик, & Недужа, 2021). The detail of the information obtained about the SSS of the foundations and the surrounding soil allows for drawing a conclusion regarding its further operation.

Even during the static analysis, the possibility of several scenarios of the train position relative to the support and the span structure is taken into account. Therefore, the obtained SSS values for the static problem allow for assessing the structure under study not in one state, but in a certain spectrum of states.

The formulation of the dynamic problem of the train impact study is more complicated than the static one, since the calculation area (pier, foundation, surrounding soil and part of the railway embankment) has significant dimensions, and the increase in the dimensionality of the problem makes it impossible to solve it on a personal computer (Zhan, Wang, Yan, Deng, Zhu, & Liu, 2022). The complexity is also noted when choosing a set of loads, since the position of the train impact changes, and this change affects the SSS of both the embankment and the pier.

Purpose

The purpose of the scientific research is to conduct numerical and modal analyses of the railway bridge pier with a variation of load combinations and to determine the strength parameters and dynamic characteristics (shape and frequency of natural vibrations).

Methodology

Pile foundations are the most economical and effective for bridge construction. The main structural elements of the pile foundation are piles. In this case, the pile foundation of the overpass pier is constructed on bored piles (Fig. 1).

The foundation is constructed with a buried pile cap. The number of piles in the foundation is 4. The pile foundation is constructed mainly in sands. To prevent collapse of these soils, metal casing pipes are immersed.

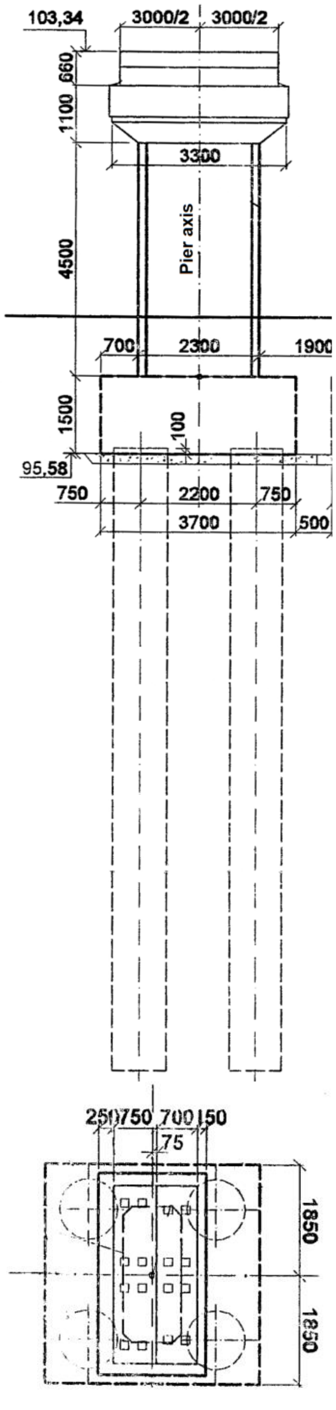


Fig. 1. General view of the pier

Installation is performed in separate sections of different lengths, which are connected to each other by welding.

To connect the piles with the pile cap, reinforcing frames are installed, which are embedded into

ISSN 2413-6212 (Online), ISSN 2227-1252 (Print)
Мости та тунелі: теорія, дослідження, практика, 2025, № 27

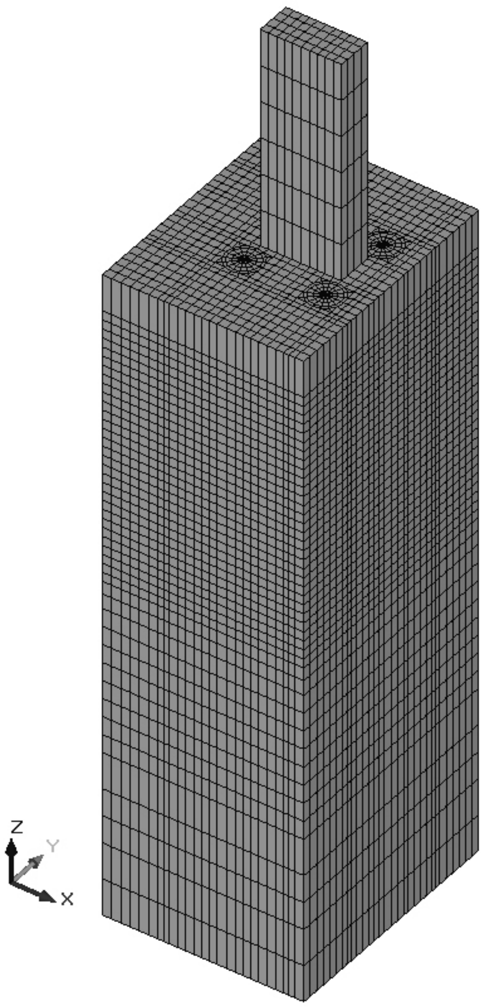
the pile body and the pile cap. The reinforcement of the pile shaft is made of two separate reinforcing frames 9.2 m in length long and a frame 4.6 m long. Concrete of C30/35 class is used for concreting the piles. The total length of the pile under the support being studied is 12.0 m (11.88 m – the level after the sludge removal). The distance between the rows of piles is 2.2 m. The pile is embedded into the pile cap by 0.1 m. To ensure the embedment of piles, their upper ends are demolished, the reinforcement is exposed and extended 1 m into the concrete of the pile cap.

The piles are connected by a pile cap, which ensures the coupled operation of the piles. The dimensions of the pile cap are as follows: length and width – 3.7 m, height – 1.5 m. Reinforcing meshes are made of longitudinal and transverse rods A400S Ø 32 mm from 35ГС steel (structural steel with medium carbon content, significant amounts of manganese and silicon). On the surface of the pile cap (edge of the pile foundation) the pier body is erected.

The connection between the pile cap and the pier body is achieved by means of reinforcing bar dowels located on the top of the pile cap. The length of the outlets is 1 m. Fixation of their position during concreting is ensured by conductors.

To reproduce the features of the pile foundation of the pier, a spatial model was used (Kelesoglu, & Springman, 2011; Khodair, & Hassiotis, 2013), which most accurately reflects its interaction with the surrounding massif (Fig. 2) (Карпиловский, Криксунов, Перельмутер, & al., 2000).

a)



b)

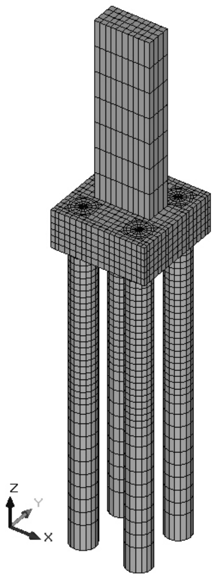


Fig. 2. FE-model and calculation scheme of the pile foundation:
a) general view; b) fragment of the model – system “support – pile cap – piles”

ISSN 2413-6212 (Online), ISSN 2227-1252 (Print)
Мости та тунелі: теорія, дослідження, практика, 2025, № 27

The height of the model is taken from considerations of the joint operation of the piles and the surrounding massif, which is composed of wet sands of different fractions of high compressibility. The data on the deformation characteristics of the soils correspond to the materials of engineering and geological research. The surrounding soil massif around the foundation is modeled using hexahedral finite elements with compatible nodes (Stewart, Jewell, & Randolph, 1993; Whelan, Gangone, Janoyan, & Jha, 2009).

To calculate the pile foundation of the pier, an elastic formulation of the problem was used, since plastic deformation of water-saturated sands does not occur (ДБН В.1.2-2:2006, 2006; ДБН В.2.1-10-2009, 2009), therefore, the elastic formulation of the problem is justified (Wu, Jiang, & Liu, 2020). The number of finite elements is 45 044, the number of nodes is 46 484 (the number of degrees of freedom is 139 452). The problem is considered a problem of medium dimension. Based on the above data, calculations of the pile foundation and the pier for 3 load combinations were performed.

Findings

The following are the main parameters of the stress state of the overpass pier foundation for the combinations. To save space, the stress components for the first combination of loads are given (Fig. 3), but the text presents the results of the analysis for all 3 load combinations.

a) b) c)

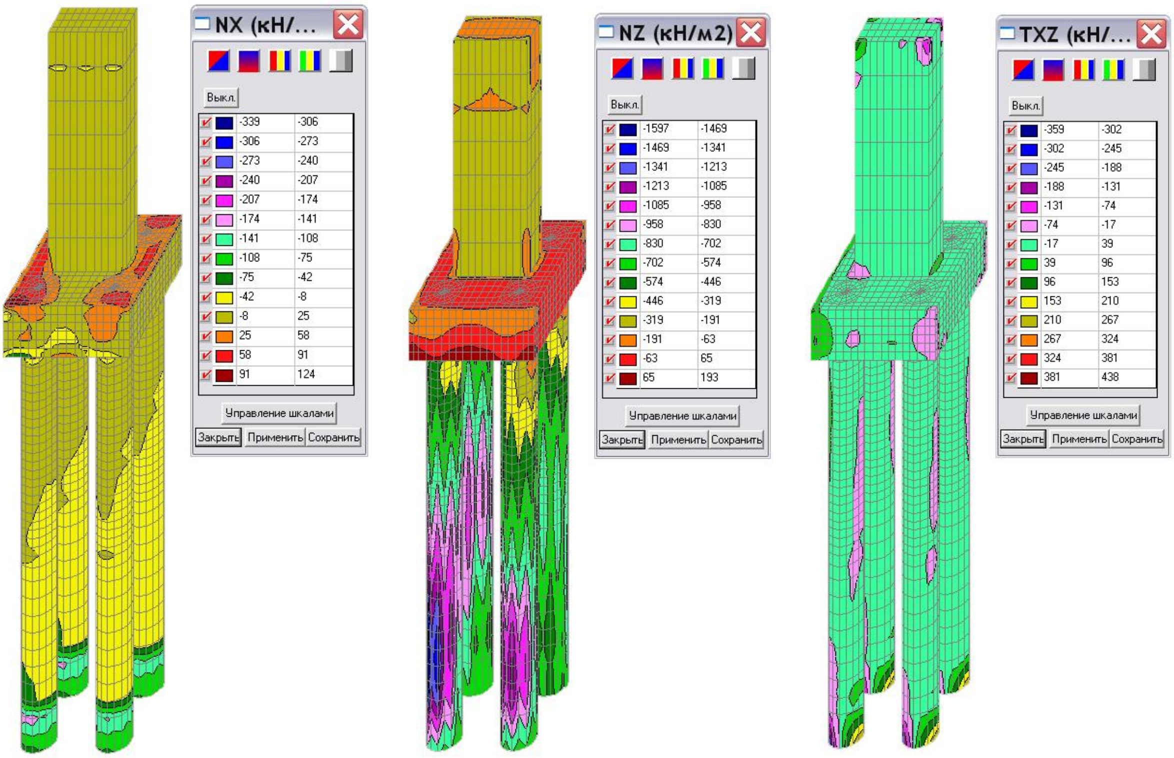


Fig. 3. Isolines and isofields of stresses in a fragment of the pile foundation model (first combination):
a) normal stresses along the horizontal axis; b) normal stresses along the vertical axis; c) tangential in the vertical plane

The parameters of the deformed state are not given, since some difficulties arose during their presentation, which are associated with the exceptional homogeneity of the displacement fields and their insignificant gradient within the calculation area (a fraction of a millimeter). Since the displacement values fully comply with the requirements of the standards, only the stressed state is analyzed.

The most stressed elements of the pile foundation under the action of their own weight are piles, and the concentration of normal vertical stresses occurs in the lower part of the pile, which is natural. Having analyzed the values of all the given components, it should be noted that component analysis is sufficient without the use of equivalent

ISSN 2413-6212 (Online), ISSN 2227-1252 (Print)
Мости та тунелі: теорія, дослідження, практика, 2025, № 27

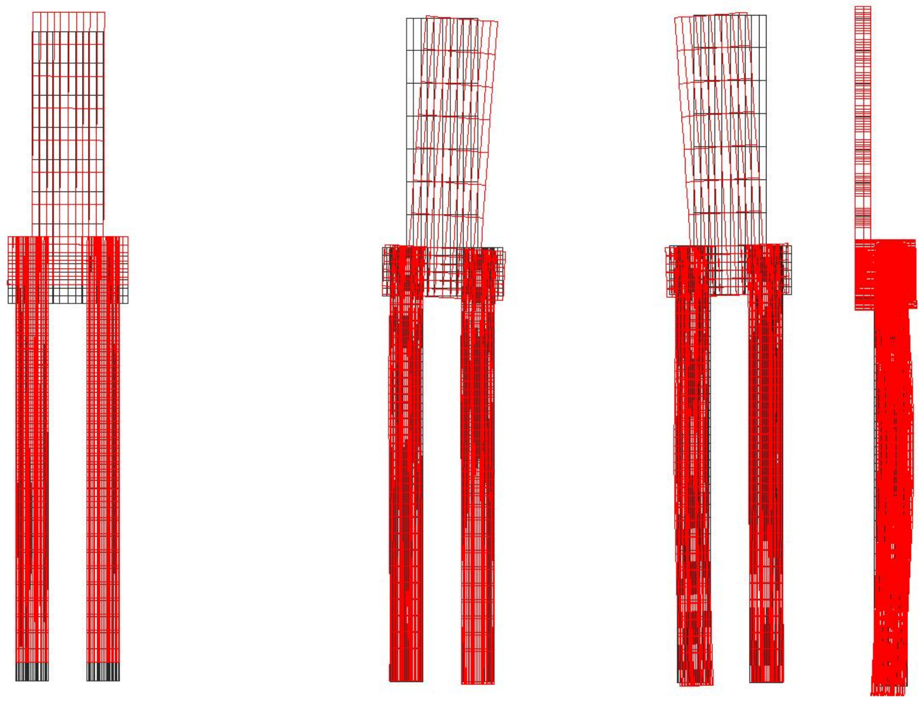
stresses, since the maximum stresses (vertical) are 1.41 … 1.54 MPa, which, compared with the calculated compressive strength R_b (R_b=[σ]=21 MPa for a concrete pile of C30/35 class), allows us to indicate a safety margin of 14.

The stress state patterns between the combinations change slightly: for combination 1, the values of maximum stresses (vertical) are 1.47 … 1.60 MPa (safety margin – 13 times); for combination 2, the values of maximum stresses (vertical) are 1.84 … 2.0 MPa (safety margin – 10.5 times); for combination 3, the values of maximum stresses (vertical) are 1.78 … 1.93 MPa (safety margin – 11 times). Thus, the pile material, concrete of C30/35 class, fully withstands all types of loads presented by the combinations.

The stress state patterns in the pile cap are as follows: for 1st combination, the values of maximum stresses (vertical) are 1.10 … 1.20 MPa (safety margin – 14 times, with the calculated compressive strength R_b=[σ]=17 MPa for the concrete pile cap of C25/30 class); for 2nd combination, the values of maximum stresses (vertical) are 1.40 … 1.5 MPa (safety margin – 11.3 times); for 3rd combination, the values of maximum stresses (vertical) are 1.41 … 1.54 MPa (safety margin – 11 times). Thus, the pile cap material, that is class C25/30 concrete, fully withstands all types of loads represented by the combinations.

For a final conclusion on the strength of the “pile – pile cap” system, it is necessary to analyze the connection region of the piles and the pile cap, as it is a stress concentrator. The stress state patterns at the place of attachment of piles and pile cap are as follows: for 1st combination, the values of maximum stresses (vertical) are 1.86 … 2.0 MPa (safety margin – 8.5 times, with the calculated compressive strength R_b=[σ]=17 MPa for the grillage of class C25/30 concrete); for 2nd combination, the values of maximum stresses (vertical) are 2.1 … 2.3 MPa (safety margin – 7.4 times); for 3rd combination, the values of maximum stresses (vertical) are 2.2 … 2.4 MPa (safety margin – 7 times). Thus, the connection region of piles and pile cap of class C25/30 concrete fully withstands all types of loads represented by the combinations.

The result of the analysis is the conclusion about the sufficient bearing capacity of all parts of the “pile – pile cap” system for all considered types of combinations with a maximum stress of 2.4 MPa (third combination, place of connection of piles and pile cap) and the corresponding safety margin of 7 times, which indicates normal operation in the future provided unchanged state of engineering and geological conditions and loads.



mode 1, frequency – 4.2 Hz mode 2, frequency – 10.7 Hz mode 3, frequency – 14.1 Hz

Fig. 4. Schemes of natural forms of vibrations with their corresponding frequencies

Fig. 4 shows the modes of natural vibrations of the foundation of a bridge-type overpass, which is isolated from the general model in the presentation graphics mode, and is shown upscaled for a more detailed analysis. 4 higher forms of free vibrations of the pile foundation of the bridge overpass are obtained. The nature of the obtained forms reflects its dynamic operation, but a significant gap between the frequencies of the first and second modes (4.2 and 10.7 Hz, respectively) indicates a possible loss of the vibration mode that develops relative to a different axis of symmetry.

Analyzing the modes and frequencies of both half-models, it should be noted that the fundamental tone (the first mode with a frequency of 4.2 Hz) was present in both models, which is quite natural. The vibrations according to the first mode are linear movements along the vertical axis without any bending; the second mode (8.2 Hz) is characterized by significant bending of the pier body in the longitudinal direction and some movements of the pile cap and piles along the vertical axis. The similar third mode (10.7 Hz) is also characterized by bending, but in a different plane, perpendicular to the second mode; the most informative from the point of view of its comparison with the frequencies of vibration-induced liquefaction of sands is the fourth mode (13.8 Hz, figure not shown), which is a partial mode of piles (a partial mode is the mode of only the corresponding part of the structure, for example, piles, pile cap or pier body).

Comparison of the frequency of 13.8 Hz with the liquefaction frequencies of wet sands (30 … 50 Hz) shows that the support's own vibrations cannot cause vibration-induced liquefaction, but for verification, a dynamic calculation of the harmonic load of the train impact should also be performed. Comparison of the frequencies of the train's harmonic vibrations (up to 20 Hz) in the presence of a significant influence of the support mass, which somewhat dampens the vibrations, can indicate that liquefaction will not occur. The fifth mode (14.1 Hz, figure not shown) is somewhat similar to the fourth mode, but is characterized by significant movements of the pier body in contrast to it; the sixth mode (17.4 Hz, figure not shown) is a form of linear movements. All frequency values are not liquefaction frequencies for wet sands.

It should be noted that the modal analysis did take into account the attached soil masses of the surrounding massif, which reduced the vibration frequency by 1 … 2 Hz. Thus, the frequencies and modes of the pile foundation without the influence of the attached soil masses are approximately as follows: 1st mode (fundamental tone) – frequency 3.2 Hz (according to both models); 2nd mode – frequency 7.2 Hz (according to the second model); 3rd mode – frequency 9.7 Hz (according to the first model); 4th mode – frequency 12.8 Hz (according to the second model); 5th mode – frequency 13.1 Hz (according to the first model); 6th mode – frequency 16.4 Hz (according to the second model).

Originality and practical value

The originality of the study lies in obtaining the values of the stresses of the overpass pier in the static formulation, as well as the dynamic characteristics (shape and frequency of natural vibrations) in the dynamic formulation. The practical value lies in the substantiated assessment of the values of the pier's vibration frequencies, which is based on the results of modal analysis and indicates the absence of sand liquefaction.

Conclusions

An analysis of the results of the stress-strain state of the overpass pier foundations was performed, taking into account the train load, which made it possible to obtain a conclusion about the high bearing capacity of all parts of the "pile – pile cap" system for all considered types of load combinations with a safety margin of 8 and 7 times, respectively, which indicates normal operation in the future provided an unchanged state of engineering and geological conditions and loads.

A modal analysis of the foundations of the overpass pier models was performed and the frequencies and modes of the foundation were obtained, which are equal to 4.2 Hz (fundamental tone) … 17.4 Hz, and from comparing these frequencies with the liquefaction frequencies of wet sands (30 … 50 Hz) it is clear that the support's own vibrations cannot cause vibration liquefaction.

REFERENCES

Dubinchyk, O., Bannikov, D., Kildieiev, V., & Kharchenko, V. (2020). Geotechnical analysis of optimal parameters for foundations interacting with loess area. *E3S Web of Conferences, II International Conference Essays of Mining Science and Practice*, 168, 00024. DOI:

https://doi.org/10.1051/e3sconf/202016800024

Kelesoglu, M. K., & Springman, S. M. (2011). Analytical and 3D numerical modelling of full-height bridge abutments constructed on pile foundations through soft soils. *Computers and Geotechnics*, *38*(8), 934-948. DOI: https://doi.org/10.1016/j.compgeo.2011.07.011

Khodair, Y., & Hassiotis, S. (2013). Numerical and experimental analyses of an integral bridge. *International Journal of Advanced Structural Engineering*, 5, 1-12. DOI: https://doi.org/10.1186/2008-6695-5-14

Lu, P., Xie, X., & Shao, C. (2012). Experimental study and numerical analysis of a composite bridge structure. *Construction and Building Materials*, 30, 695-705. DOI: https://doi.org/10.1016/j.conbuildmat.2011.11.008

Stewart, D. P., Jewell, R. J., & Randolph, M. F. (1993). Numerical modelling of piled bridge abutments on soft ground. *Computers and Geotechnics*, 15(1), 21-46. DOI: https://doi.org/10.1016/0266-352X(93)90015-Y

Whelan, M. J., Gangone, M. V., Janoyan, K. D., & Jha, R. (2009). Real-time wireless vibration monitoring for operational modal analysis of an integral abutment highway bridge. *Engineering Structures*, 31(10), 2224-2235. DOI: https://doi.org/10.1016/j.engstruct.2009.03.022

Wu, L., Jiang, G., & Liu, X. (2020). Analytical and Numerical Analysis of Additional Stress in Foundation of Bridge Approach Embankment. *Geotechnical and Geological Engineering*, 38, 6069-6082. DOI: https://doi.org/10.1007/s10706-020-01414-2

Zhan, J., Wang, C., Yan, Y., Deng, L., Zhu, W., & Liu, J. (2022). Modal analysis and condition evaluation of substructures for simply supported high-speed railway bridge based on a simplified model. *Journal of Bridge Engineering*, 27(8), 04022058. DOI: https://doi.org/10.1061/(ASCE)BE.1943-5592.0001900

Дубінчик, О. І., & Недужа, Л. О. (2021). Обґрунтування напружено-деформованого стану пальового фундаменту з використанням програмних комплексів. *Мости та тунелі: теорія, дослідження, практика*, 20, 13-18. DOI: https://doi.org/10.15802/bttrp2021/245247

ДБН В.2.1-10-2009 (2009). *Основи та фундаменти споруд. Основні положення проєктування*. Київ: Мінрегіонбуд України.

ДБН В.1.2-2:2006 (2006). *Навантаження і впливи. Норми проектування*. Київ: Мінбуд України.

Карпиловский, В. С., Криксунов, Э. З., Перельмутер, А. В., & al. (2000). *SCAD для пользователя*. Киев: ВВП «Компас».

Загоруйко, Є. А., & Лозовий, С. О. (2012). Імітаційне моделювання пальових фундаментів з метою визначення їх деформацій. *Нові технології в будівництві*, 1-2, 68-73.

О. Л. ТЮТЬКІН[1], О. І. ДУБІНЧИК[2*], ЦЮАНЬ ЧЖАН[3], КАЙЦЯНЬ ДУ[4]

[1]Кафедра «Транспортна інфраструктура», Український державний університет науки і технологій, вул. Лазаряна, 2, Дніпро, Україна, 49010, тел. +38 (066) 290 45 18, ел. пошта o.l.tiutkin@ust.edu.ua, ORCID 0000-0003-4921-4758

[2*]Кафедра «Транспортна інфраструктура», Український державний університет науки і технологій, вул. Лазаряна, 2, Дніпро, Україна, 49010, тел. +38 (095) 582 14 88, ел. пошта o.i.dubinchyk@ust.edu.ua, ORCID 0000-0003-4059-2357

[3]Інженерно-архітектурний інститут, Професійно-технічний університет будівництва Чжецзян Гуаньша, Дун'ян, Чжецзян, Китайська народна республіка, 322100, тел. +86-0579-86668872, ел. пошта 55846673@qq.com, ORCID 0009-0009-4691-9913

[4] Інженерно-архітектурний інститут, Професійно-технічний університет будівництва Чжецзян Гуаньша, Дун'ян, Чжецзян, Китайська народна республіка, 322100, тел. +86-0579-86668872, ел. пошта dukaiqian1@163.com, ORCID 0009-0007-4977-5842

ЧИСЕЛЬНИЙ ТА МОДАЛЬНИЙ АНАЛІЗ ПРОМІЖНОЇ ОПОРИ ЗАЛІЗНИЧНОГО МОСТУ

Мета. Метою наукового дослідження є проведення чисельного та модального аналізів проміжної опори залізничного мосту з варіацією сполучень навантажень та визначенням параметрів міцності та динамічних характеристик (форми і частоти власних коливань). **Методика.** Найбільш економічними та ефективними для будівництва мостів є пальові фундаменти. У даному випадку пальовий фундамент проміжної опори шляхопроводу споруджується на буронабивних палях. Для відтворення особливостей пальового фундаменту проміжної опори застосовано просторову СЕ-модель, яка найбільш точно відображає його взаємодію із оточуючим масивом. **Результати.** Виконано аналіз результатів напружено-деформованого стану фундаментів проміжної опори шляхопроводу із урахуванням поїзного навантаження, який надав змогу отримати висновок про високу несучу здатність усіх частин системи «палі – ростверк» на всі розглянуті види сполучень із запасом міцності в 8 і 7 разів відповідно, що свідчить про нормальну експлуатацію в подальшому при незмінному стані інженерно-геологічних умов і навантажень. Місце кріплення паль та ростверку бетон класу С25/30 повністю витримує усі види навантажень, представлених сполученнями. Проведений модальний аналіз фундаментів моделей проміжної опори шляхопроводу й отримані частоти і форми фундаменту, які дорівнюють 4,2 Гц (основний тон)…17,4 Гц, а із порівняння цих частот з частотами розрідження вологих пісків (30…50 Гц) зрозуміло, що власні коливання опори не можуть викликати вібраційного розрідження. **Наукова новизна.** Наукова новизна дослідження полягає в отриманні значень напружень проміжної опори в статичній постановці, а також динамічних характеристик (форми і частити власних коливань) в динамічній постановці. **Практична значимість.** Практична значимість полягає в обґрунтованій оцінці величин частот коливань проміжної опори, що базується на результатах модального аналізу і свідчить про відсутність розрідження пісків.

Ключові слова: залізничний міст; проміжна опора; чисельний аналіз; модальний аналіз; форми і частоти власних коливань

REFERENCES

Dubinchyk, O., Bannikov, D., Kildieiev, V., & Kharchenko, V. (2020). Geotechnical analysis of optimal parameters for foundations interacting with loess area. *E3S Web of Conferences, II International Conference Essays of Mining Science and Practice*, 168, 00024. DOI: https://doi.org/10.1051/e3sconf/202016800024 (in English)

Kelesoglu, M. K., & Springman, S. M. (2011). Analytical and 3D numerical modelling of full-height bridge abutments constructed on pile foundations through soft soils. *Computers and Geotechnics*, *38*(8), 934-948. DOI: https://doi.org/10.1016/j.compgeo.2011.07.011(in English)

Khodair, Y., & Hassiotis, S. (2013). Numerical and experimental analyses of an integral bridge. *International Journal of Advanced Structural Engineering*, 5, 1-12. DOI: https://doi.org/10.1186/2008-6695-5-14 (in English)

Lu, P., Xie, X., & Shao, C. (2012). Experimental study and numerical analysis of a composite bridge structure. *Construction and Building Materials*, 30, 695-705. DOI: https://doi.org/10.1016/j.conbuildmat.2011.11.008 (in English)

Stewart, D. P., Jewell, R. J., & Randolph, M. F. (1993). Numerical modelling of piled bridge abutments on soft ground. *Computers and Geotechnics*, 15(1), 21-46. DOI: https://doi.org/10.1016/0266-352X(93)90015-Y (in English)

Whelan, M. J., Gangone, M. V., Janoyan, K. D., & Jha, R. (2009). Real-time wireless vibration monitoring for operational modal analysis of an integral abutment highway bridge. *Engineering Structures*, 31(10), 2224-2235. DOI: https://doi.org/10.1016/j.engstruct.2009.03.022 (in English)

Wu, L., Jiang, G., & Liu, X. (2020). Analytical and Numerical Analysis of Additional Stress in Foundation of Bridge Approach Embankment. *Geotechnical and Geological Engineering*, 38, 6069-6082. DOI: https://doi.org/10.1007/s10706-020-01414-2 (in English)

Zhan, J., Wang, C., Yan, Y., Deng, L., Zhu, W., & Liu, J. (2022). Modal analysis and condition evaluation of substructures for simply supported high-speed railway bridge based on a simplified model. *Journal of Bridge Engineering*, 27(8), 04022058. DOI: https://doi.org/10.1061/(ASCE)BE.1943-5592.0001900 (in English)

Dubinchyk, O. I., & Neduzha, L. O. (2021). Obgruntuvannia napruzheno-deformovanoho stanu palovoho fundamentu z vykorystanniam prohramnykh kompleksiv. *Mosty ta tuneli: teoriia, doslidzhennia, praktyka*, 20, 13-18. DOI: https://doi.org/10.15802/bttrp2021/245247 (in Ukrainian)

DBN V.2.1-10-2009 (2009). *Osnovy ta fundamenty sporud. Osnovni polozhennia proiektuvannia*. Kyiv: Minrehionbud Ukrainy. (in Ukrainian)

DBN V.1.2-2:2006 (2006). *Navantazhennia i vplyvy. Normy proektuvannia*. Kyiv: Minbud Ukrainy. (in Ukrainian)

Karpilovskiy, V. S., Kriksunov, E. Z., Perelmuter, A. V., & al. (2000). SCAD dlya polzovatelya. Kiev: VVP «Kompas».(in Russian)

Zahoruiko, Ye. A., & Lozovyi, S. O. (2012). Imitatsiine modeliuvannia palovykh fundamentiv z metoiu vyznachennia yikh deformatsii. *Novi tekhnolohii v budivnytstvi*, 1-2, 68-73. (in Ukrainian)

Надійшла до редколегії 21.04.2025.
Прийнята до друку 22.05.2025.