МОСТИ ТА ТУНЕЛІ: ТЕОРІЯ, ДОСЛІДЖЕННЯ, ПРАКТИКА

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CORRESPONDENCE OF NUMERICAL MODELS OF LANDSLIDE-PRONE SLOPE TO PHYSICAL OBJECTS OF RESEARCH

Purpose. The purpose of this work is to substantiate the adequacy of numerical models to real physical objects, which significantly depends on how precisely the physical and mechanical characteristics of the soil massif are determined. This especially applies to the determination of critical geometric parameters of slopes. **Methodology.** Generalization of well-known research in geomechanics and geotechnics, which used an approach to determine the main mechanical characteristics of the soil massif based on the analysis of the deformed state, including critical, real objects, and subsequent back-calculations. **Findings**. According to the methodology in accordance with the specified approach, all objects under investigation, regardless of their final purpose, are classified according to a justified classification feature. For each class, the following basic real objects are selected, on which irreversible changes in the deformation state have occurred. If the geological structure, hydrogeology of the rock massif, external influences, as well as the geometric parameters of such a destructive process, which is a landslide, are known, then through reverse calculations it is possible to find such averaged physical and mechanical characteristics, the use of which for the appropriate class of selected objects will allow to build and investigate an adequate geomechanical model. **Originality**. A methodical approach to the determination of physical and mechanical characteristics by their selection, using cases of manifestations of the stress-strain state of pre-classified real objects, is proposed. **Practical value.** Determining the critical conditions of natural slopes and artificial slopes based on the assessment of the physical and mechanical characteristics of the soil massif as a whole by means of reverse calculations will allow to safely design the construction of buildings and structures on the slopes and ensure the reliable operation of open pit mining operations.

Keywords: geomechanical models; soil ledge; slope; stress-strain state; natural balance; stability; slope processes; adequacy

Introduction

Methodological developments aimed at increasing the reliability of research in geomechanics and geotechnics regularly appear in the domestic and foreign scientific space. Famous research by Crouch S. geomechanical models based on the limite element method (Crouch, & Starfield, 1987). The fundamentals of studying deformations in solids as basic characteristics when creating numerical models are outlined in the works of J.F. Bell (Bell, 1984). The solution to the problem of loss of stability of complex structural objects from the simultaneous action of external and internal factors is considered in the works of Gilmore R. (Gilmore, 1981).

A large complex of works related to the substantiation of the adequacy of geomechanical models was carried out by a team of scientists under the leadership of O. Shashenko, O. Sdvyzhkova both in a deterministic setting (Солодянкин, & Сдвижкова, 2003; Martovitsky, Khalimendik, & Sdviyzhkova, 2011; Шашенко, Солодянкін, & Мартовицький, 2012) and on the basis of probabilistic approaches (Шашенко, Тулуб, & Сдвижкова, 2002). A large overview of information sources addressing this problem is given in the monograph (Ковров, Собко, Азюковський, et al., 2020).

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Purpose

Computer modeling has become an integral part of geomechanical and geotechnical research. With the help of appropriate software packages, this approach allows obtaining faster, less expensive and more accurate results than physical modeling. The problem of this approach remains the justification of the adequacy of numerical models to real physical objects. This is especially true for determining the stress field parameter, which in real objects can only be estimated by measured deformations and only for an elastic stress-strain state. In this regard, the development of methodological approaches to the justification of numerical models in geomechanics and geotechnics, aimed at bringing theoretical and practical results closer together, is an actual task.

Methodology

The problem of assessing the stability of a homogeneous soil slope using the finite element method for the case of violation of its stability is given in the work (Шашенко, 2009).

We consider a soil slope (ledge) with a height H composed of homogeneous rock with a volume weight γ and having an angle of inclination to the horizon α, its upper part with height H_{90} has vertical stability, the compressive strength of the rocks R_c, and the rocks themselves are noticeably weakened due to internal defects, which is taken into account by the coefficient of structural weakening k_c (Fig. 1).

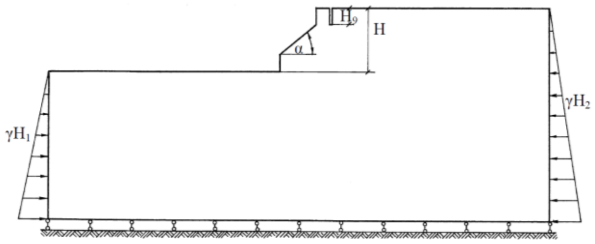

Fig. 1. Calculation scheme for solving the problem of stability of a homogeneous soil slope (Шашенко, Сдвижкова, Хозяйкина, & Полищук, 2005)

For a given height of such a ledge and with its known shape, it is necessary to determine such physical and mechanical characteristics at which the location and shape of the sliding line of the collapse prism coincide with the same parameters that were found in natural conditions.

The object of consideration is a real case of slope collapse in a quarry, which occurred at one time (Шашенко, Сдвижкова, Хозяйкина, & Полищук, 2005). For this case, the geometric parameters of the ledge are known.: $H = 43{,}0$ m, slope angle $\alpha = 22^0$, the height of the vertical part of the slope H_{90}, as well as the shape and location parameters of the slip curve. It is necessary to select such physical and mechanical properties of rocks at which the shape and characteristics of the slip curve in nature and in numerical experiment coincide with sufficient accuracy. Six numerical models were studied, of which the greatest agreement between the slip line parameters occurred with the following physical and mechanical parameters: volume density $\gamma = 1900$ kg/m^3; modulus of elasticity $E = 2{,}78 \cdot 10^{-9}$ kg/m^2, Poisson's ratio $\mu = 0{,}2$, uniaxial compressive and tensile strength $R_c = 297 \cdot 10^4$ kg/m^2, $R_p = 135 \cdot 10^4$ kg/m^2 respectively.

To approximate the study area (see Fig. 1), quadrangular elements were used (Fig. 2). In the course of solving the problem, all components of the stress-strain state of the medium were determined at the nodes of the finite element mesh and at the center of gravity of each element.

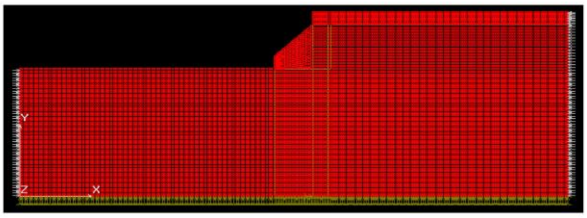

Fig. 2. Approximation of the study area by quadrangular elements

The main prerequisite for determining a possible shear surface was the following hypothesis: the shear surface is the geometric location of the points at which the destruction of the material occurred under the combined influence of normal and tangential stresses.

The ratio proposed by O. M. Shashenko was used as a strength criterion (Шашенко, Сдвижкова, Хозяйкина, & Полищук, 2005):

$$\left(\sigma_1 - \sigma_3\right)^2 - R_c^2\psi - \left(1-\psi\right)R_c\left(\sigma_1 + \sigma_3\right) = 0, \qquad (1)$$

where σ_1, σ_3 – the highest and lowest tension, respectively, $\psi = R_p / R_c$ – fragility factor equal to the ratio of the tensile strength R_p – fragility factor equal to the ratio of the compression strength R_c.

The expression for equivalent tensions (Шашенко, 2009) is as follows:

$$\sigma_e = \frac{(\psi - 1)(\sigma_1 + \sigma_2) + \sqrt{(1-\psi)^2 (\sigma_1 + \sigma_3)^2 + 4\psi(\sigma_1 - \sigma_3)^2}}{2\psi} \le R_c k_c . \quad (2)$$

The equivalent tension is some stress reduced to a uniaxial state, in other words, a stress equivalent to the uniaxial.

The difference between the strength of the rock mass and the strength of soil samples is estimated by the coefficient of structural weakening, obtained on the basis of the statistical theory of strength with the assumption that the strength of rock samples is distributed according to the normal law (Голуб, & Полищук, 2000). For the conditions under consideration, when only the statistical heterogeneity of the medium without cracks was taken into account, the structural weakening coefficient $k_c = 0,4$.

Taking into account the heterogeneity of rocks, the condition of the limit state of rocks takes the form:

$$\sigma_e = R_c k_c , \quad (3)$$

or

$$k = \frac{R_c k_c}{\sigma_e} = 1 . \quad (4)$$

In the future, we will call this value k the safety factor of the soil relative to the point of the soil slope under consideration.

Thus, the problem is reduced to finding such a boundary surface on which condition (1) is satisfied. In the numerical solution of the problem of plane deformation of the considered section of the rock mass, the result of the calculations is the trace of this surface, i.e. the set of points of the plane at which the limit relation (2) is satisfied. With respect to the finite element method, this is a set of elements in which the combination of normal and shear tensions satisfies the strength conditions (1) with some predetermined accuracy.

Findings

The results of the modelling of the problem described above are shown in Fig. 3. Elements in which this ratio is fulfilled are shown in black, that is, the safety factor k is close to one. The centers of gravity of these elements form an isoline of equivalent tensions $\sigma_e = R_c k_c$, that is, an isoline of the strength coefficient $k = 1$.

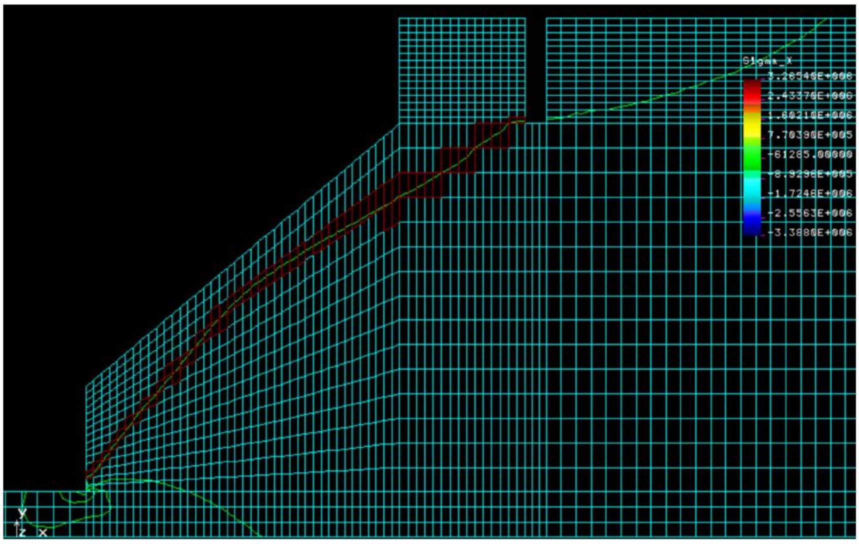

Fig. 3. The picture of the coincidence of the distribution of the components of normal tensions with the elements in which the condition is satisfied (4)

The isoline has a convex-concave shape, begins at the foot of the bench and ends at the place where a technological cut is made, facilitating the displacement of the soil mass. The shape of the obtained shear line is close to that actually observed in natural conditions and meets the sliding conditions according to V. V. Sokolovsky. Thus, for a class of homogeneous slopes with horizontal laying of soil and similar physical and mechanical characteristics, it is possible to use the studied digital model, built on the finite element method.

The second example of using the method of constructing a geomechanical model based on the «reverse method» is given in the work of M.P. Kropotkin. The simulated slope had a height of 43 m, the distance from the edge to the water level in the river was 116 m. The soil slope is composed of 9 calculated geological elements (CGE), lying horizontally, the model parameters are given in Table 1. The power of each element and the slope contour are shown in Fig. 4.

Table 1

Model parameters of calculated geological elements

Model parameters of soils at the stage II (the emergence and development of deep creep)								
№ of layer	Angle of internal friction, degrees		Specific adhesion, ts/m^2		Density, t/m^3			Effective porosity, relative unit
	above groundwater level	in a water-saturated state	above groundwater level	in a water-saturated state ***	above groundwater level	in a water-saturated state	taking into acc. weighting effect	
1	20	15,0	1,0	0,8/0,2	1,80	1,93	0,95	0,20
2*	-	10,0	-	3,2/0,8	-	1,70	0,76	0,04
3	-	6,0/7,5**	-	3,1	-	1,74	-	0,03
4	19	-	4,3	-	2,10	-	-	0,08
5	31	30,0	0,7	0,6	1,86	-	1,02	0,20
6	-	18,5	-	3,6/1,2	-	1,92	-	0,05
7	-	29,0	-	0,8	-	-	0,01	0,20
8	-	12,0	-	3,0/2,0	-	1,90	-	0,04
9	-	7,4	-	4,4/4,2	-	1,75	-	0,02

Notes:
* The characteristics of CGE-2 are given in the table for its "tongue" part. In the rest of the part, the density is accepted as equal 1,73 t/m^3.
** Numerator - for the horizontal zone of preliminary displacements; denominator - for the rest CGE -3.
*** Numerator - structural adhesion C_c; denominator - cohesion of connectivity C_w.

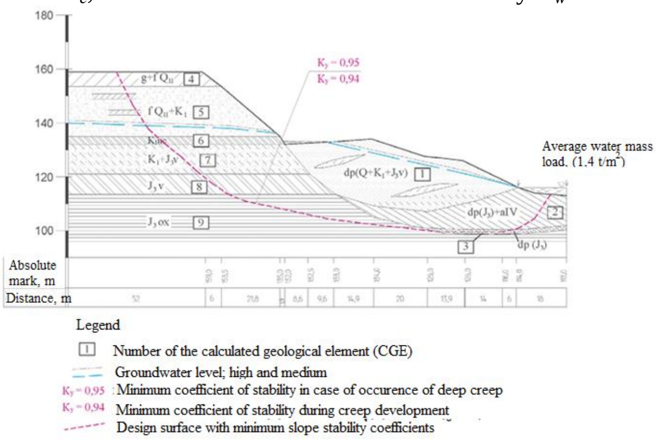

Fig. 4. Modeling of slope sliding

The calculations were performed using the PSK software package block, which is based on a mathematical model in the form of a system of limit equilibrium equations reflecting the relationship between shear and holding forces in the soil massif. The model is considered in a two-dimensional formulation taking into account the interaction forces acting along the edges of the calculated sections and corresponding to the initial stress-strain state immediately before the start of the sliding process.

The physical and mechanical characteristics of the CGE, on the basis of which the modeling was carried out, are given in Table 1. The actual shape of the displacement surface was determined based on the results of inclinometric observations. Mathematical modeling and natural studies showed a fairly close match of the results. Thus, for displacements of this type with a similar geological structure of the massif and hydrogeological situation, the physical and mechanical characteristics can be taken as close to those given in Table 1. The given example can be used to check the adequacy of software packages based on other physical principles.

A large number of such hydrogeological models have been constructed and studied under the supervision of Professor I. O. Sadovenko in the study of hydrogeological regimes occurring within layered rock massifs weakened by underground workings of coal mines. Mathematical numerical modeling affected the processes of geofiltration in a rock mass heterogeneous in capacity and permeability, taking into account the relationship between underground and surface waters, their flow through low-permeability layers, as well as changes in boundary conditions and geofiltration parameters in time and space. When constructing hydrogeological models, the «reverse method» was usually used, which allowed obtaining adequate modeling results (Sadovenko, Zagrytsenko, Podvigina, & Dereviagina, 2016, 2017; Sadovenko, Zahrytsenko, Podvigina, Dereviahina, & Brzeźniak, 2018; Bazaluk, Sadovenko, Zahrytsenko, et al., 2021; Sadovenko, Tymoshchuk, Zahrytsenko, et al., 2024).

Originality and practical value

The analysis of information sources on mathematical modeling of geotechnical systems has been conducted, which has shown that one of the difficult stages of the study is the transfer of the results of laboratory tests of samples to the entire soil massif, which consists of an artificial slope or a natural slope. If such a massif is layered, then this task is significantly complicated. One of the ways to solve the problem of the adequacy of the results of mathematical modeling to natural measurements is to divide the objects of study into classes, search for cases of loss of stability of the classification elements and assess the physical and mechanical characteristics of the object as a whole by selecting them in such a way that the modeling results coincide with the results of natural measurements. This approach is called «inverse modeling». Examples of such modeling are considered. It has been proven that this approach in a number of cases gives fairly accurate results, which makes the forecast a reliable tool for designing and ensuring the stability of future buildings, structures and elements of technology.

Conclusions

An analysis of methodological approaches to the construction and study of geomechanical models was carried out, as a result of which an «reverse method» was proposed, which allows, with known boundary parameters, to establish the patterns of distribution of the stress-strain state of soil massifs, including the boundary state, and groundwater filtration.

Geomechanical models developed for specific cases that have shown sufficient agreement between analytical and natural results can be used as reference models when testing software products based on other physical conditions and under other strength conditions.

It has been proven that on the basis of such geomechanical models, it is possible to correctly develop appropriate technological solutions for the construction of slopes and the safe operation of quarries.

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ВІДПОВІДНІСТЬ ЧИСЛОВИХ МОДЕЛЕЙ ЗСУВОНЕБЕЗПЕЧНОГО СХИЛУ ФІЗИЧНИМ ОБ'ЄКТАМ ДОСЛІДЖЕНЬ

Мета. Метою даної роботи є обґрунтування адекватності числових моделей реальним фізичним об'єктам, яка суттєво залежить від того, наскільки точно визначені фізико-механічні характеристики ґрунтового масиву. Особливо це торкається визначенню критичних геометричних параметрів укосів або схилів. **Методика.** Узагальнення відомих досліджень у геомеханіці і геотехніці, які використовували підхід до визначення основних механічних характеристик ґрунтового масиву на підставі аналізу деформованого стану, у тому числі критичного, реальних об'єктів, і подальшого виконання зворотних розрахунків. **Результати**. Згідно з методикою відповідно до означеного підходу всі об'єкти, що досліджуються, незалежно від їх кінцевого призначення, класифікуються за обґрунтованою класифікаційною ознакою. Для кожного класу підбираються такі базові реальні об'єкти, на яких відбулися незворотні зміни деформаційного стану. Якщо відомі геологічна будова, гідрогеологія породного масиву, зовнішні впливи, а також геометричні параметри такого руйнівного процесу, яким є зсув, то шляхом зворотних розрахунків можна знайти такі усереднені фізико-механічні характеристики, використання яких для відповідного класу виділених об'єктів дозволить побудувати і дослідити адекватну геомеханічну модель. **Наукова новизна.** Запропоновано методичний підхід до визначення фізико-механічних характеристик шляхом їх підбору, використовуючи випадки граничних проявів напружено-деформованого стану попередньо класифікованих реальних об'єктів. **Практична значимість.** Визначення критичних станів природних схилів і штучних укосів на основі оцінки фізико-механічних характеристик ґрунтового масиву в цілому шляхом зворотних розрахунків дозволить безпечно проєктувати будівництво будівель і споруд на схилах балок та забезпечити надійну експлуатацію відкритих гірничих робіт.

Ключові слова: геомеханічні моделі; ґрунтовий уступ; схил; напружено-деформований стан; природна рівновага; стійкість; зсувні процеси; адекватність

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