МОСТИ ТА ТУНЕЛІ: ТЕОРІЯ, ДОСЛІДЖЕННЯ, ПРАКТИКА

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NUMERICAL ANALYSIS OF REINFORCEMENT OF SECTIONS WITH TRANSITIONAL RIGIDITY ON APPROACHES TO RAILWAY BRIDGES

Purpose. To analyze the mechanism of work of embankments, to model three-dimensional models and to compare methods of their reinforcement on transitional areas on approaches to railway bridges using the Lira software based on finite element method (FEM). To do comprehensive analysis of results and comparison of constructive performance of different reinforcement methods. **Methodology.** Numerical finite element modeling (FEM) had been performed to study the work of embankments in transition sections on approaches to railway bridges. Four three-dimensional models of the construction of the transitional section, corresponding to the real bridge, had been constructed. These models had been tested for nominal load to evaluate their performance relative to each other. **Findings.** As a result of the calculations, the deformation characteristics for the basic model and each of the reinforcement types had been obtained, and their comparisons had been made by determining the maximum vertical deformations and vertical deformations at key points. The analyze of feasibility of using the tested reinforcement methods under different real initial conditions had been performed. **Originality.** The main aspects of the work of the transition sections over long periods of time had been revealed. Comparisons of fundamentally different methods and types of reinforcement of sections with transient stiffness had been made. The expediency of the methods of reinforcement of the embankment construction in the transitional sections that had been proposed in previous works had been tested. **Practical value.** The proposed reinforcement methods may be used depending on the specific design conditions, the budget, and other factors. Evaluating the work of different types of reinforcement during numerical analysis makes it possible to move away from large-scale field trials and focus on other methods, which significantly reduces time spent on solving current problems.

Keywords: stress-strain state; numerical modeling; finite element method; transitional stiffness; approaches; bump; dip

Introduction

The problems of excessive vertical deformations in transition sections on the approaches to the railway bridges have arisen almost from the very beginning of history of appearance of artificial structures on the railways. However, greater attention had been paid to it amplification in the 20th century, and the largest studies of the mechanics of their work came at the end of the 20th – beginning of the 21st century (Nicks, 2009b).

Studies of arias with transient stiffness have shown the need to reinforcement them for normal operation of the structure over long periods of time. The analysis of areas with transient stiffness performed in previous work has allowed not only to outline the main causes of their occurrence, but also to investigate and compare the methods of their reinforcement in developed countries (Marochka, & Boboshko, 2018).

While the methods of reinforcement based on the complete restructuring of the embankment mound are the most widely used by foreign engineers, among the patents of Ukrainian bridge-builders there are more frequent ways of reinforcement, which require a minimum amount of excavation, and sometimes do not even involve dismantling the upper structure of the track (Tejada A. de Miguel, 2015; Marochka, & Boboshko, 2018).

The main criteria for choosing a reinforcement method are to evaluate the complexity of the erection, the financial feasibility and the design efficiency. If the first two criteria are easily computational, the last one requires the greatest amount of research.

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The choice of the most rational from the point of view of the right design reinforcement can be determined by various methods, among which the basic are mathematical modeling, experimental research and full-scale tests (Seo, 2003; Paixao, 2014).

Mathematical modeling is the least complex from an organizational and financial point of view and allows research to be carried out regardless of natural and geopolitical conditions. Besides, the reliability of the results can be verified by experimental method or field tests.

The determination of the stress-strain state allows us to focus on the deformations arising in the structure due to its long-term use.

When using mathematical modeling by finite element method, there are several factors to consider: constructive correctness of the model, real conditions of structure attachment, and the load application that will meet the necessary research conditions (static long-term or dynamic shock load) (Paixao, & Fortunato, 2009).

Purpose

To construct the models of transitional sections, which would correspond to the real work of the structure and to compare their reinforcements on the basis of the obtained results.

Methodology

To solve the problem of studying the interaction of the embankment of the bridge in the transition section with its reinforcement and to select the most appropriate reinforcement, we use a complex method, which includes the analysis of stress-strain state (SSS) in mathematical modeling. This approach to this problem makes it possible to find out the dependencies or regularities of the formation of the SSS of construction with different variants of reinforcement of transition sections, since the mathematical modeling results reveals sufficient or insufficient correspondence of the proposed models to the real bridge (Тяпочкин, 2009).

Numerical analysis using finite element method had been used for mathematical modeling of the task of interaction of reinforcement with the surrounding mass. While not giving information on the nature of the method described in many classical analytical works, it should be noted that FEM is the most progressive method of numerical simulation, since it allows to study complex deep and underground structures in interaction with the surrounding array. Analyzing a number of advantages of the given method, it should be noted that in solving FEM problems, no additional assumptions are required to be introduced into the calculation scheme (FE model), i.e. the influence of uncertain parameters on the calculation results is minimal (Купрій, & Тютькін, 2012; Купрій, Кулаженко, & Гудкова, 2015).

To determine the stress-strain state of the structure of the transition section on the approach to the railway bridge and to determine the most optimal method of reinforcement of the embankment, three-dimensional finite-element models were constructed in the LIRA software complex (Rahmani, & Kebdani, 1981; Петраков, 1995; Dahlberg, 2001; Nicks, 2009a).

The experimental model, that had been used in this work based on a real railway bridge on the Loshkarovka – Pavlopillya line of the Merefa – Kherson line in Ukraine.

Since the deformations with respect to the XoZ plane are symmetrical, the model is executed for half the construction of the transition section. When modeling the initial structure, the general model of length 44.5 m includes the section of the embankment 14.5 m in front of the abutment, the abutment itself, and 20 m of the embankment behind the abutment (Figure 1). The latter is the main object of observation.

Three types of reinforcement of the embankment construction were selected for verification and comparison: 1) reinforcement by gabion boxes; 2) reinforcement by vertical soil cement piles; 3) the use of reinforced and sorted soils.

The soil array had been modeled from hexagonal and octagonal finite elements, preferably with a side of 0.5 m.

The main points of observation are the upper points of contact of abutment and embankment. For simplicity, they were given the names "T1", "T2" and "T3" (Figure 2).

To simulate the structural reinforcement of gabion boxes (model № 2), the top part of the embankment had been given a stiffness corresponding to the gabion boxes. Reinforcement of the structure is carried out by means of pairs of gabion boxes of sizes 3000×2500×2000, 3000×2500×1500 and 3000×2500×1000. Gabions are modeled as octagonal finite elements of appropriate stiffness.

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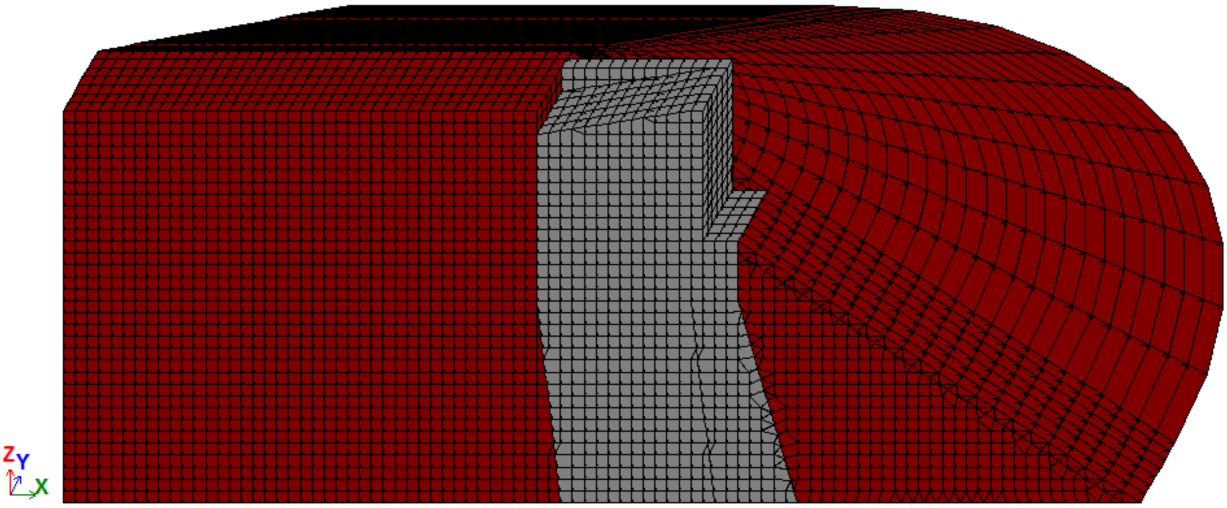

Fig. 1. Finite element model № 1

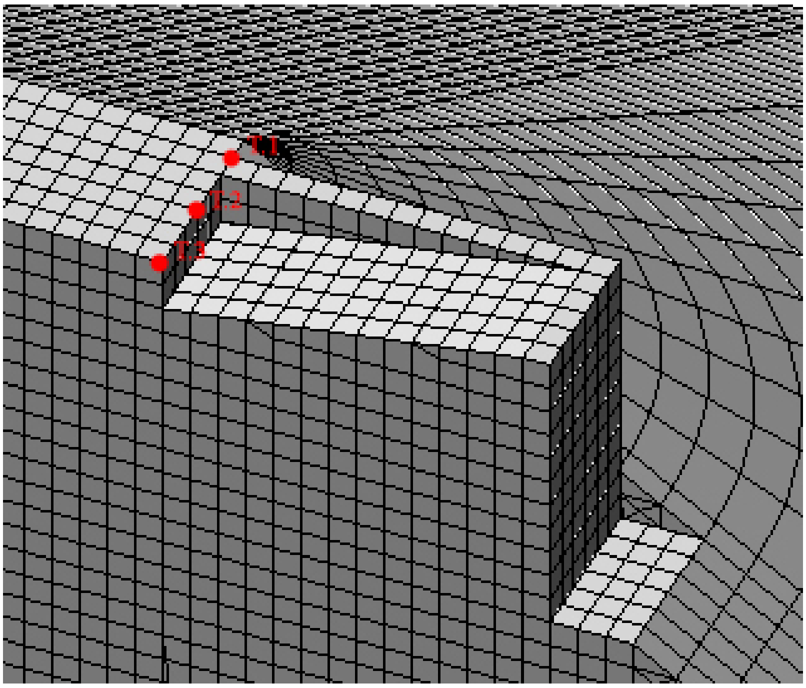

Fig. 2. Points of deformation reading

For model № 3 (reinforcement with soil cement piles), the decision was made to model piles with hexagons in section, consisting of finite elements in the form of triangular prisms.

Piles of length from 13 to 8 m with multiplicity of 1 m are placed along the "X" axis by a cascade of 3 pieces for each length. The distance between the axis of the first pile and the abutment is 2 m, the distance between the piles is 1 m in the axes. The distance from the pile's axes to the edge of the model is 0.5 m.

Model № 4 is the most widely used type of reinforcement in the EU countries and provides for the replacement of the transition embankment soil by sorted and cement reinforced soils. The model is constructed in the same way as model № 1 with the replacement of the stiffness of the soil of the transitional section to those corresponding to the sorted (blue color) and reinforced cement sorted soil (pink color).

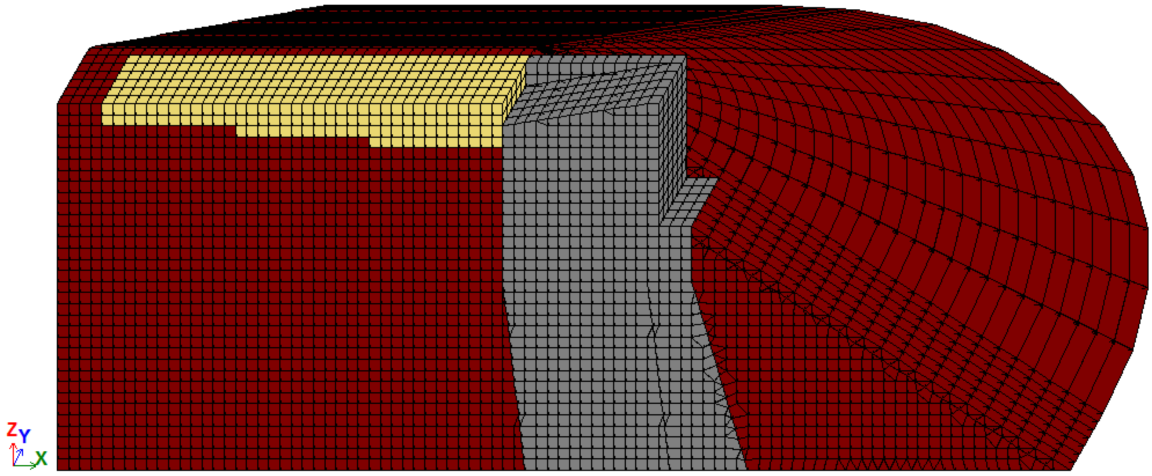

Fig. 3. Finite element model № 2

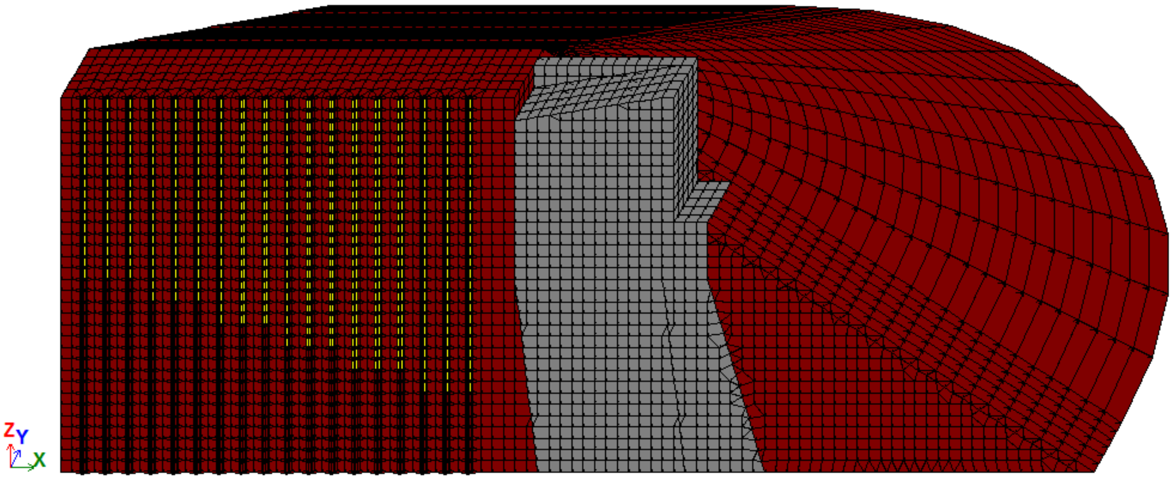

Fig. 4. Finite element model № 3

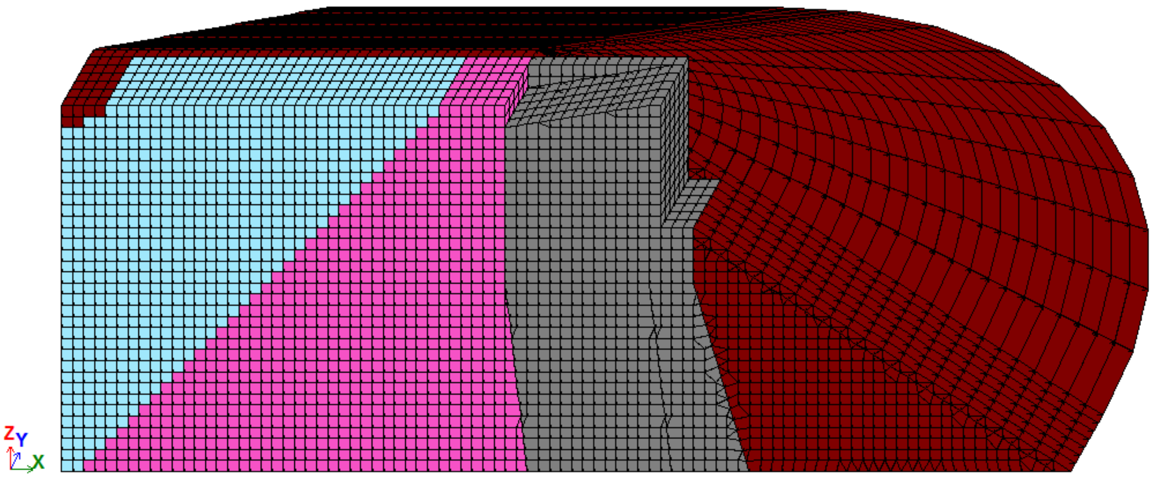

Fig. 5. Finite element model № 4

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To further calculations of the joint work of the abutment and the surrounding array using finite elements corresponding to the transitional sections on the railway bridges, soil layers and additional reinforcement elements, the characteristics of the material stiffness were assigned, which numerically reflect their deformation characteristics (modulus of elasticity and Poisson's ratio), and the magnitude of the own weight of the materials (Карпиловский, Криксунов, Перельмутер, & al., 2000).

The characteristics of the above parameters are given in Table 1.

The variants were calculated on a nominal vertical distributed load of 10 kN/m^2 applied to the upper embankment plane behind the abutment.

Table 1

Finite element characteristics

Number of stiffness type	Item name	E, kN/m^3	ρ, kN/m^3	μ
Soils				
1	Loam	25000	20.1	0.3
2	Sorted Loam	80000	22.3	0.3
3	Sorted reinforced Loam	1.5×10^5	23.1	0.3
Reinforced concrete				
4	B25	3.0×10^7	25.0	0.2
Other materials				
5	Soil cement	2.5×10^5	24.5	0.3
6	Gabion boxes	3.0×10^5	20.0	0.3

Results

The results of the calculation show real reflections of deformations of the soil massif of the embankment. In calculations № 1, 2, 3 and 4, the results of the vertical displacements of the structure, which are shown in Figures 6, 7, 8 and 9, respectively, had been obtained.

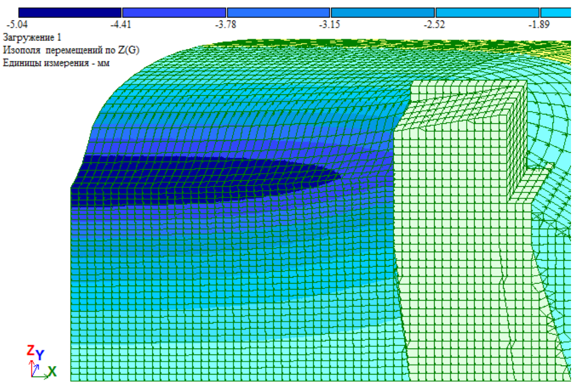

Fig. 6. Vertical displacement diagram, scheme № 1

As can be seen in Figure 6, the maximum deformations in the initial structure of the transition section were 5.04 mm, the deformations at the points T1, T2 and T3 – 2.53; 3.9 and 4.17 mm respectively.

With the reinforcement of the upper section of the structure by gabions, the maximum vertical displacement decreased by 65.9 %, and the displacement at points T1, T2 and T3 was 0.98; 1.11 and 1.15 mm respectively, up 61.2; 71.5 and 72.4 % less.

With the reinforcement of the structure by soil cement piles, the maximum vertical displacement decreased by 47.4 %, and the displacement at points T1, T2, and T3 was 1.4; 2.13 and 2.25 mm respectively, up 44.7; 45.4 and 46 % less.

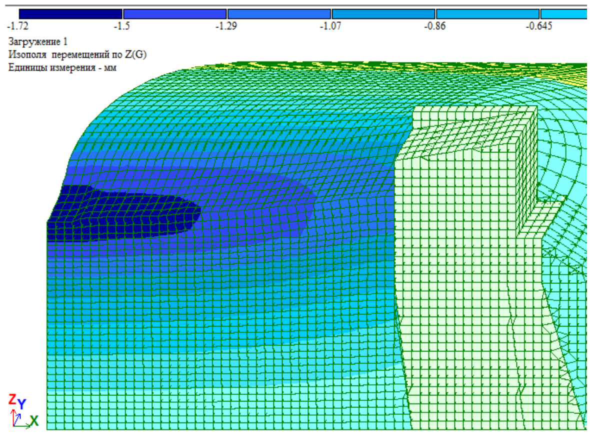

Fig. 7. Vertical displacement diagram, scheme № 2

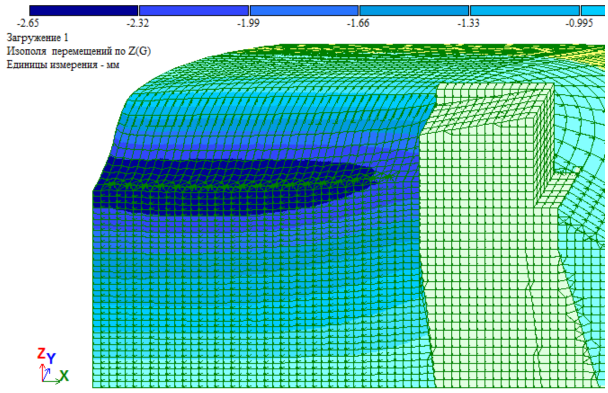

Fig. 8. Vertical displacement diagram, scheme № 3

With the reinforcement of the structure by sorted and cemented soils, the maximum vertical displacement decreased by 69.2 %, and the displacement at points T1, T2 and T3 was 0.77; 0.8 and 0.81 mm respectively by 69.7; 79.5 and 80.6 % less.

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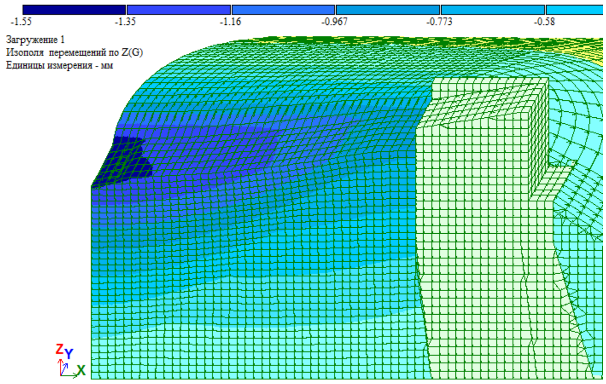

Fig. 9. Vertical displacement diagram, scheme № 4

A general comparison of vertical relocation of key points is summarized in Table 2.

Table 2

Comparison of vertical displacements

Model №	Δ_{max}	Δ_{T1}	Δ_{T2}	Δ_{T3}
№ 1 The original design	5.04	2.53	3.90	4.17
№ 2 Gabion boxes	1.72	0.98	1.11	1.15
№ 3 Soil cement piles	2.65	1.40	2.13	2.25
№ 4 Reinforced and sorted soils	1.55	0.77	0.80	0.81

As a result of the tests, the greatest reinforced effect was shown in models № 2 and № 4. The reinforcement № 3 though showed the result, but it was lower than expected.

From a technological point of view, Reinforcement № 3 is the easiest to implement because it does not require disassembly of the upper structure of the coil, and, as a result, allows you to reduce cost not only for the work on reinforcement, but also significantly reduce the loss of profit from stopping traffic on the railway section.

Reinforcement № 2, although needing to dismantle the upper structure, is not costly in terms of the use of materials and construction machinery and the delivery of materials, as gabions can be assembled directly on site in the design position in the prepared ditches.

Reinforcement № 4, although it showed the best result, is the costliest in the planned bridge reinforcement. Such a reinforcement scheme is most appropriate for the construction of new structures or renovations. This method is the most widely used in the EU countries.

As a conclusion, from the technological and practical point of view, in the current conditions of the Ukrainian railways and the manufacturing sector, the most expedient method is to reinforce the transitional areas with gabions.

Reinforcement with soil cement piles is a local temporary solution that can increase the stiffness of the embankment structure and thereby reduce subsidence.

Reinforcement of sorted and cemented soils, though yielding the best results, is the costliest and expedient only in the construction of new bridges.

Originality and practical value

When calculating soil arrays and their interaction with other objects, the finite element method gives the closest picture to the real conditions of the nature of deformations. Its use makes it possible to solve the set tasks in the shortest possible time, taking into account the detail of the built models, which brings them as close as possible to the real mechanics of the construction.

The design of reinforcing of transitional sections using soil cement piles is the least costly and allows for reinforcement without disassembly of the top track structure.

As an alternative to this reinforcement method, it would be advisable to propose the use of gabion structures. This method does not require the high cost of material delivery, reinforcement, and maintenance of the consignment over a medium period of time.

The most effective method in the long run is the use of sorted and cemented soils in the construction of embankment structures.

Conclusions

On the basis of the conducted researches it is possible to draw conclusions about the effectiveness of different methods of strengthening the structures of sections with a transient stiffness index from each other.

An analysis of the stress-strain state of the structure makes it possible to draw conclusions from the weakest points of the transition areas and to visually check the feasibility of any of the proposed reinforcements by finite element modeling.

Each of the proposed reinforcement methods is situational and can be used depending on the design conditions and the budget laid down.

REFERENCES

Dahlberg, T. (2001). Some railroad settlement models – a critical review. *Proceedings of the Institution of Mechanical Engineers*, 215 (F), 289-300.

Nicks, J. (2009a). Preliminary numerical simulations of the bump at the end of the railway bridges. *Fourth International Young Geotechnical Engineers Conference*.

Nicks, J. E. (2009b). *The bump at the end of the railway bridge*. Doctoral dissertation, Texas A&M University.

Marochka, V. V., & Boboshko, S. H. (2018). Analysis of the problems of sections with the transitional rigidity indicator in world bridging. *Мости та тунелі: теорія, дослідження, практика*, 16, 82-92.

Marochka, V. V., & Boboshko, S. H. (2018). Development of technology of arranging areas with transitional stiffness index on approaches to railway bridges. *Мости та тунелі: теорія, дослідження, практика*, 13, 99-106.

Paixao, A. (2014). *Transition zones in railway tracks. An experimental and numerical study on the structural behaviour*. Ph.D. thesis, LNEC & FEUP.

Paixao, A. & Fortunato, E. (2009). *Novas solucoes de superstrutura de via para a alta velocidade*. ITC informacao cientifica-Laboratyrio Nacional de Engenharia Civil.

Seo, J. B. (2003). *The bump at the end of the bridge: An investigation*. Doctoral dissertation, Texas A&M University. Texas A&M University.

Rahmani, O., & Kebdani, S. (1981). *Introduction à la Méthode des Eléments Finis pour les Ingénieurs*. O.P.U. Alger.

Tejada A. de Miguel (2015). *Analysis of the problems associated with dynamic interaction between train, track and structure at transition zones*. Doctoral dissertation, E.T.S.I. Caminos, Canales y Puertos (UPM).

Карпиловский, В. С., Криксунов, Э. З., Перельмутер, А. В., & al. (2000). *SCAD для пользователя*. Киев: ВВП «Компас».

Купрій, В. П., Кулаженко, Є. Ю., & Гудкова, А. С. (2015). Моделювання сумісної роботи конструкції кріплення котловану та ґрунту з застосуванням методу скінчених елементів (МСЕ). *Мости та тунелі: теорія, дослідження, практика*, 7, 19-26.

Купрій, В. П., & Тютькін, Д. В. (2012). Моделювання напружено-деформованого стану кріплення глибокого котловану. *Мости та тунелі: теорія, дослідження, практика*, 3, 89-94.

Тяпочкин, А. В. (2009). Горизонтальные перемещения армогрунтовых насыпей. *Испытания и расчёты конструкций транспортных сооружений. Научные труды ОАО ЦНИИС*, 251, 69-74.

Петраков, А. А. (1995). *О расчетных моделях нелинейно-деформируемого грунтового массива*. Полтава: ПТУ.

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ЧИСЕЛЬНИЙ АНАЛІЗ ПІДСИЛЕННЯ ДІЛЯНОК З ПЕРЕХІДНИМ ПОКАЗНИКОМ ЖОРСТКОСТІ НА ПІДХОДАХ ДО ЗАЛІЗНИЧНИХ МОСТІВ

Мета. Вивчення роботи насипів, моделювання об’ємних моделей та порівняння методів їх підсилення на перехідних ділянках на підходах до залізничних мостів за допомогою програмного комплексу Lira, що базується на методі скінчених елементів (МСЕ). Комплексний аналіз результатів та порівняння конструктивної ефективності роботи різних методів підсилення. **Методика.** Для вивчення роботи насипів на перехідних ділянках на підходах до залізничних мостів проведено числове моделювання методом скінчених елементів (МСЕ). Побудовано чотири тривимірні моделі конструкції перехідної ділянки, що відповідає реальному мосту. Виконане випробування цих моделей на номінальне навантаження для оцінки їх ефективності одна відносно одної. **Результати.** За результатом розрахунків було отримано деформативні характеристики для базової моделі та кожного з типів підсилення, проведено їх порівняння шляхом визначення максимальних вертикальних деформацій та вертикальних деформацій в ключових точках.

Виконано аналіз доцільності використання випробуваних методів підсилення за різних реальних вихідних умов. **Наукова новизна.** Виявлено основні аспекти роботи перехідних ділянок протягом великих проміжків часу. Виконано порівняння принципово різних за методами влаштування та типами підсилень ділянок з перехідним показником жорсткості. Перевірена доцільність методів підсилення конструкції насипу в перехідних ділянках, запропонованих в попередніх роботах. **Практична значимість.** Запропоновані варіанти підсилення можуть використовуватися в залежності від специфічних умов конструкції, закладеного бюджету та інших факторів. Оцінка роботи різних типів підсилення в ході чисельного аналізу дає змогу відійти від масштабних натурних випробувань та зосередитися на інших методах, що дозволяє істотно знизити часові витрати на рішення актуальних проблем.

Ключові слова: деформований стан; метод скінчених елементів; перехідна жорсткість; підходи; швидкісний рух; передмостова яма

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ЧИСЛЕННЫЙ АНАЛИЗ УСИЛЕНИЯ УЧАСТКОВ С ПЕРЕХОДНЫМ ПОКАЗАТЕЛЕМ ЖЕСТКОСТИ НА ПОДХОДАХ К ЖЕЛЕЗНОДОРОЖНЫМ МОСТАМ

Цель. Изучение работы насыпей, моделирование объемных моделей и сравнение методов их усиления на переходных участках на подходах к железнодорожным мостам с помощью программного комплекса Lira, который базируется на методе конечных элементов (МКЭ). Анализ результатов и сравнение конструктивной эффективности работы различных методов усиления. **Методика.** Для изучения работы насыпей на переходных участках на подходах к железнодорожным мостам проведено численное моделирование методом конечных элементов (МКЭ). Построено четыре трехмерные модели конструкции переходного участка, которые соответствуют реальному мосту. Выполнено испытание этих моделей на номинальную нагрузку для оценки их эффективности друг относительно друга. **Результаты.** По результатам расчетов было получено деформативные характеристики для базовой модели и каждого из типов усиления, проведено их сравнение путем определения максимальных вертикальных деформаций и вертикальных деформаций в ключевых точках. Выполнен анализ целесообразности использования испытанных методов усиления при различных реальных исходных условий. **Научная новизна.** Выявлено основные аспекты работы переходных участков в течение больших промежутков времени. Выполнено сравнение принципиально различных по методам устройства и типам усилений участков с переходным показателем жесткости. Проверена целесообразность методов усиления конструкции насыпи в переходных участках, предложенных в предыдущих работах. **Практическая значимость.** Предложенные варианты усиления могут использоваться в зависимости от специфических условий конструкции, заложенного бюджета и других факторов. Оценка работы различных типов усиления в ходе численного анализа позволяет отойти от масштабных натурных испытаний и сосредоточиться на других методах, что позволяет существенно снизить временные затраты на решение актуальных проблем.

Ключевые слова: деформированное состояние; метод конечных элементов; переходная жесткость; подходы; скоростное движение; предмостовая яма

REFERENCES

Dahlberg, T. (2001). Some railroad settlement models – a critical review. *Proceedings of the Institution of Mechanical Engineers*, 215 (F), 289-300. (in English)

Nicks, J. (2009a). Preliminary numerical simulations of the bump at the end of the railway bridges. *Fourth International Young Geotechnical Engineers Conference*. (in English)

Nicks, J. E. (2009b). *The bump at the end of the railway bridge*. Doctoral dissertation, Texas A&M University. (in English)

Marochka, V. V., & Boboshko, S. H. (2018). Analysis of the problems of sections with the transitional rigidity indicator in world bridging. *Mosty ta tuneli: teorija, doslidzhennja, praktyka*, 16, 82-92. (in English)

Marochka, V. V., & Boboshko, S. H. (2018). Development of technology of arranging areas with transitional stiffness index on approaches to railway bridges. *Mosty ta tuneli: teorija, doslidzhennja, praktyka*, 13, 99-106. (in English)

Paixao, A. (2014). *Transition zones in railway tracks. An experimental and numerical study on the structural behaviour*. Ph.D. thesis, LNEC & FEUP. (in English)

Paixao, A. & Fortunato, E. (2009). *Novas solucoes de superstrutura de via para a alta velocidade*. ITC informacao cientifica-Laboratyrio Nacional de Engenharia Civil. (in English)

Seo, J. B. (2003). *The bump at the end of the bridge: An investigation*. Doctoral dissertation, Texas A&M University. Texas A&M University. (in English)

Rahmani, O., & Kebdani, S. (1981). *Introduction à la Méthode des Eléments Finis pour les Ingénieurs*. O.P.U. Alger. (in English)

Tejada A. de Miguel (2015). *Analysis of the problems associated with dynamic interaction between train, track and structure at transition zones*. Doctoral dissertation, E.T.S.I. Caminos, Canales y Puertos (UPM). (in English)

Karpilovskij, V. S., Kriksunov, Je. Z., Perel'muter, A. V., & al. (2000). *SCAD dlja pol'zovatelja*. Kiev: VVP «Kompas». (in Russian)

Kuprij, V. P., Kulazhenko, Je. Ju., & Ghudkova, A. S. (2015). Modeljuvannja sumisnoji roboty konstrukciji kriplennja kotlovanu ta gruntu z zastosuvannjam metodu skinchenykh elementiv (MSE). *Mosty ta tuneli: teorija, doslidzhennja, praktyka*, 7, 19-26. (in Ukrainian)

Kuprij, V. P., & Tjutjkin, D. V. (2012). Modeljuvannja napruzheno-deformovanogho stanu kriplennja ghlybokogho kotlovanu. Mosty ta tuneli: teorija, doslidzhennja, praktyka, 3, 89-94. (in Ukrainian)

Tjapochkin, A. V. (2009). Gorizontal'nye peremeshhenija armogruntovyh nasypej. *Ispytanija i raschjoty konstrukcij transportnyh sooruzhenij. Nauchnye trudy OAO CNIIS*, 251, 69-74. (in Russian)

Petrakov, A. A. (1995). *O raschetnyh modeljah nelinejno-deformiruemogo gruntovogo massiva*. Poltava: PTU. (in Russian)

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